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CONSTRAINTS ON THE STEADY-STATE R-MODE AMPLITUDE IN NEUTRON STAR TRANSIENTS

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ABSTRACT

Recent observations suggest that neutron stars in low-mass X-ray binaries rotate within a narrow range of spin frequencies clustered around 300 Hz. A proposed explanation for this remarkable fact is that gravitational radiation from a steady-state r-mode oscillation in the neutron star's core halts the spin-up due to accretion. For the neutron star transients, balancing the time-averaged accretion torque with gravitational wave emission from steady-state, constant amplitude r-mode pulsations implies a quiescent luminosity too bright to be consistent with observations (in particular of Aql X-1). The viscous dissipation (roughly 10 MeV per accreted nucleon for a spin of 300 Hz) from such an r-mode makes the core sufficiently hot to power a thermal luminosity $\sim 10^{34}\,\mathrm{erg\,s^{-1}}$ when accretion halts. This is the *minimum* quiescent luminosity that the neutron star must emit when viscous heating in the core is balanced by radiative cooling from the surface, as is the case when the core of the star is superfluid. We therefore conclude that either the accretion torque is much less than $\dot{M}(GMR)^{1/2}$, or that a steady-state r-mode does not limit the spin rate of the neutron star transients. Future observations with *Chandra* and *XMM* promise to further constrain the amount of viscous dissipation in the neutron star core.

Subject headings: accretion, accretion disks — stars: individual(Aquila X-1) — stars: neutron — stars: oscillations — stars: rotation — X-rays: stars — gravitation — gravitational waves

1. INTRODUCTION

With the launch of *RXTE*, precision timing of accreting neutron stars has opened new threads of inquiry into the behavior and lives of these objects. The neutron stars in low-mass X-ray binaries (LMXBs) have long been thought to be the progenitors of millisecond pulsars (see Bhattacharya 1995 for a review), and a long-standing observational goal has been the detection of a spin period of a neutron star in an LMXB. Recent observations (see van der Klis 1999 for a review) have finally provided conclusive evidence of millisecond spin periods of neutron stars in about one-third of known Galactic LMXBs. Altogether, there are seven neutron stars in LMXBs with spin periods firmly established by either pulsations in the persistent emission (in the millisecond X-ray pulsar SAX J1808.4-3658; Wijnands & van der Klis 1998) or oscillations during type I X-ray bursts (so-called burst QPOs, first discovered in 4U 1728–34; Strohmayer et al. 1996). There are an additional thirteen sources with twin kHz QPOs for which the neutron star's spin may be approximately equal to the frequency difference (van der Klis 1999). A striking feature of all these neutron stars is that their spin frequencies lie within a narrow range, $260\,\mathrm{Hz} < \nu_{\mathrm{spin}} < 589\,\mathrm{Hz}$. The frequency range might be even narrower if the burst QPOs seen in KS 1731–260, MXB 1743–29, and Aql X-1 are at the first harmonic of the spin frequency, as is the case with the 581 Hz burst oscillations in 4U 1636–536 (Miller 1999). If this is the case, then the range of observed frequencies is $260\,\mathrm{Hz} < \nu_{\mathrm{spin}} < 401\,\mathrm{Hz}$. The neutron stars in LMXBs accrete at diverse rates, from $10^{-11}\,M_\odot\,\mathrm{yr}^{-1}$ to the Eddington limit, $10^{-8}\,M_\odot\,\mathrm{yr}^{-1}$. Since disk accretion exerts a substantial torque on the neutron star and these systems are very old (van Paradijs & White 1995), it is remarkable that these neutron stars' spins are so tightly correlated, and that none of the neutron stars are rotating anywhere near the breakup frequency of roughly 1 kHz.

Observations therefore suggest that neutron stars in LMXBs are somehow stuck within a narrow band of spin frequencies well below breakup. Two explanations for this convergence of spin frequencies have been proffered. White & Zhang (1997) argued that the magnetospheric spin equilibrium model (see Ghosh & Lamb 1979 and references therein), which is applicable to the accreting X-ray pulsars, is also at work in LMXBs. In this scenario, the neutron star's magnetic field ($B \sim 10^9$ G) dominates accretion near the stellar surface, and the Keplerian period at the magnetospheric radius roughly equals the spin period, so that the accretion stream exerts no net torque on the star. Because the sources' luminosities (and presumably accretion rates) vary by several orders of magnitude, White & Zhang (1997) noted that this explanation requires either that the accretion rate be tightly correlated with the neutron star's magnetic field, $B \propto \dot{M}^{1/2}$, or that the torque be roughly independent of accretion rate when the magnetospheric radius approaches the radius of the neutron star. Moreover, the persistent pulses typical of magnetic accretors must also be hidden most of the time.

The other class of theories, first considered by Papaloizou & Pringle (1978) and Wagoner (1984), invoke the emission of gravitational radiation to balance the torque supplied by accretion. Bildsten (1998) proposed that equilibrium between the accretion torque and gravitational radiation can explain the narrow range of observed spin frequencies. The source for the gravitational radiation could be a mass quadrupole formed by misaligned electron capture layers in the neutron star's crust (Bildsten 1998). Alternatively, as proposed independently by Bildsten (1998) and Andersson, Kokkotas, & Stergioulas (1999), current quadrupole radiation from an unstable r-mode oscillation (Andersson 1998; Friedman & Morsink 1998) in the liquid core of the neutron star could also limit the spin, as might occur in hot, newly born neutron stars (Lindblom, Owen, &

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Morsink 1998; Owen et al. 1998; Andersson, Kokkotas, & Schutz 1999). Because the accretion rate of LMXBs does vary by several orders of magnitude, the small range of $\nu_{\rm spin}$ among these objects also requires a correlation between the quadrupole moment and accretion rate. This correlation is much less restrictive, however, than for magnetic equilibrium theories because of the steep dependence of gravitational wave torque on the spin frequency.

These theories have renewed interest in accreting neutron stars as gravitational wave sources. If gravitational radiation does in fact halt the spin-up of accreting neutron stars, then, regardless of the mechanism producing the gravitational radiation, the brightest LMXBs (such as Sco X-1, with dimensionless strain $h_c \gtrsim 2\times10^{-26}$; Bildsten 1998) are also promising sources for ground-based gravitational wave interferometers, such as LIGO, VIRGO, GEO, and TAMA (Bildsten 1998; Brady & Creighton 1999). It is not certain, however, that accreting neutron stars in LMXBs do emit gravitational radiation. The *only* evidence to date is their narrow range of spin frequencies. It is therefore important to look for astronomical observations, doable today, that can either corroborate or rule out the various mechanisms for gravitational radiation from LMXBs.

In this paper we present a new observational test for r-mode driven gravitational radiation from neutron stars in one set of LMXBs, the soft X-ray transients. These are LMXBs in which accretion outbursts, lasting for days to months, are followed by periods of quiescence, lasting on the order of years to decades. Typical time-averaged (over the recurrence interval, rather than just over the outburst) accretion rates $\langle\dot{M}\rangle$ for these sources are $\lesssim 10^{-10}\,M_\odot\,{\rm yr}^{-1}$, smaller than those in the brighter persistently accreting LMXBs. We show that the quiescent X-ray luminosities of these neutron star transients (in particular Aql X-1, which exhibits burst QPOs with a frequency 549 Hz; Zhang et al. 1998) can be used to determine whether r-modes with amplitudes sufficient to balance the accretion torque are present in their cores.

Recent theoretical (Brown, Bildsten, & Rutledge 1998) and observational (Rutledge et al. 1999b) works suggest that at least some fraction of the quiescent luminosity of a neutron star transient is thermal emission from the neutron star's surface. Motivated by the possibility of indirectly measuring the core temperature of an accreting neutron star, we consider the amount of heat that must be lost, on average, by the neutron star to maintain a thermal steady state. If the spins of neutron star transients are set by the equilibrium between the *time-averaged* accretion torque and gravitational wave emission by *steady-state* (i.e., constant amplitude) r-mode pulsations in their cores, the required amplitude of the pulsations can be computed (§ 2). The steady-state assumption implies a certain magnitude of viscous dissipation, i.e., heat deposited directly into the core of the neutron star. If the core is superfluid, Urca neutrino emission is suppressed and this heat escapes as thermal radiation from the surface of the star. We show (§ 3) that in this case the X-ray luminosity in quiescence, L_q, would be about 5–10 times greater than that observed. If the nucleons in the core are normal, then, as shown by Levin (1999), r-mode pulsations are thermally unstable (at least for saturation amplitudes of order unity). In this case it is unlikely that r-modes are currently excited in any of the known Galactic LMXBs. If for some reason a thermal steady state could be achieved in a normal fluid core, however, then Urca neutrino emission would carry away most of the r-mode heating, and the resulting lower quiescent thermal luminosities would be consistent, within uncertainties, with observations. Our test does not depend on how the r-mode is damped, but only on the assumptions that the dissipated energy is deposited into the thermal bath of the star and that the star has reached a rotational and thermal steady state. We are only inquiring into total energetics, i.e., whether the viscous heating present matches that required by the spin equilibrium with the accretion torque.

2. R-MODE VISCOUS HEATING OF ACCRETING NEUTRON STARS

Recently Andersson (1998) and Friedman & Morsink (1998) showed that gravitational radiation excites the r-modes (large scale toroidal fluid oscillations similar to geophysical Rossby waves) of rotating, *inviscid* stars. Lindblom et al. (1998) compared the gravitational wave growth timescale $\tau_{\rm gr}$ for the r-modes with the viscous damping timescale τ_v set by shear and bulk viscosities for normal fluids (i.e., no superfluidity); at rotation rates $\Omega \lesssim 0.065\Omega_K$, where $\Omega_K = (GM/R^3)^{1/2}$ is the Keplerian angular velocity at the surface of the star, the damping is sufficient to preclude unstable growth. The modes are excited, however, over a wide range of spin frequencies and temperatures that includes typical values for the neutron star transients.

Gravitational waves radiate away angular momentum at a rate

$$\left.\frac{dJ}{dt}\right|_{\rm gr} = -\frac{2J_c}{\tau_{\rm gr}}, \qquad (1)$$

where

$$J_c = -\frac{3}{2}\alpha^2\Omega\tilde{J}MR^2 \qquad (2)$$

is the canonical angular momentum of the $(l=2, m=2)$ r-mode (Friedman & Schutz 1978; Owen et al. 1998), α is the dimensionless amplitude of the mode, and $\tilde{J}$ is a dimensionless constant that accounts for the distribution of mass in the star (Owen et al. 1998). The gravitational wave growth time $\tau_{\rm gr}$ is negative, which implies instability. In a rotational steady state, this angular momentum loss is balanced by the accretion torque, $N_{\rm accr}$. For a fiducial torque, we assume that each accreted particle transfers its Keplerian angular momentum to the neutron star, with a net accretion torque $N_{\rm accr} = \dot{M}(GMR)^{1/2}$. Using $\tau_{\rm gr}$ as evaluated by Lindblom et al. (1998), we find the steady-state r-mode amplitude (Bildsten 1998; Levin 1999),

$$\alpha_{\rm steady} = 7.9\times10^{-7}\left(\frac{\dot{M}}{10^{-11}\,M_\odot\,{\rm yr}^{-1}}\right)^{1/2}\left(\frac{300\,{\rm Hz}}{\nu_{\rm spin}}\right)^{7/2}, \qquad (3)$$

such that the fiducial accretion torque $N_{\rm accr}$ is balanced by r-mode angular momentum loss $dJ/dt|_{\rm gr}$.

The gravitational radiation reaction adds energy to the unstable r-mode at a rate

$$\left.\frac{dE_c}{dt}\right|_{\rm gr} = -\frac{2E_c}{\tau_{\rm gr}}, \qquad (4)$$

where

$$E_c = \frac{1}{2}\alpha^2\Omega^2\tilde{J}MR^2 \qquad (5)$$

is the canonical energy of the $(l=2, m=2)$ r-mode (Friedman & Schutz 1978; Owen et al. 1998). In a steady state all of this energy must be dissipated by viscous processes at a rate $W_d = dE_c/dt$. In terms of the accretion luminosity, $L_A = GM\dot{M}/R = N_{\rm accr}\Omega_K$, the dissipation rate is

$$\frac{W_d}{L_A} = -\frac{1}{\Omega_K}\frac{dE_c/dt|_{\rm gr}}{dJ_c/dt|_{\rm gr}} = \frac{1}{3}\frac{\Omega}{\Omega_K}. \qquad (6)$$

The viscosity in the neutron star originates from several possible sources. For normal *npe* matter, calculations of the viscous transport coefficients exist only at near-nuclear densities (Flowers & Itoh 1979). The components of such a core are strongly degenerate, and phase-space restrictions impart a characteristic T^{-2} dependence to the shear viscosity (Cutler & Lindblom 1987). Compressing a fluid element of neutron star matter causes it to emit neutrinos as the *npe* mixture reestablishes β-equilibrium, so the bulk viscosity has an Urca-like T^6 dependence (Sawyer 1989). Another possibility for the viscosity is that it is caused by mutual friction in the neutron-proton superfluid (Mendell 1991). In this case the viscous damping is independent of temperature.

While the total amount of viscous dissipation W_d depends only on the assumption of a steady-state r-mode amplitude, the amount of heat actually deposited into the star depends on the nature of the damping. If the dominant viscous mechanism is bulk viscosity (i.e., for core temperatures $T \gtrsim 10^9\,\mathrm{K}$), then the dissipated energy is released in the form of neutrinos, which promptly leave the star. The core temperatures of LMXBs are most likely less than $10^9\,\mathrm{K}$, however, in which case the dissipation mechanism is either shear viscosity or mutual friction. For both of these mechanisms, the heat W_d is deposited directly into the core of the star; we shall assume this to be the case in the rest of this paper.

Levin (1999) first noted that, if the nucleons in the core are normal, the r-modes damped by shear viscosity are likely to be thermally unstable, at least for saturation amplitudes of order unity. The heating from the shearing motions decreases the viscosity, and so the r-mode amplitude increases, which heats the star even more. The result is a thermal and dynamical runaway. As envisaged by Levin (1999), the neutron star enters a limit cycle of slow spin-up to some critical frequency, at which the r-mode becomes unstable, followed by a rapid spin-down until the mode is once again damped. As the neutron star cools, accretion again exerts a positive torque on the star, and the cycle repeats. Because the r-modes are present at a nonzero amplitude for only $\sim 10^{-7}$ of the entire cycle's duration, it is unlikely that any of the known LMXBs harbor active r-modes *and* have normal fluid cores.

For a superfluid core, where the damping is due to mutual friction (and hence independent of temperature), the neutron star can reach a state of three-fold equilibrium (Bildsten 1998; Levin 1999): the temperature is set by the balance of viscous heating and radiative or neutrino cooling, the r-mode's amplitude is set by the balance of gravitational radiation back-reaction and viscous damping, and the spin is set by the balance of accretion torque and angular momentum loss to gravitational radiation. It is this scenario that we shall examine for existing evidence of r-mode spin regulation.

While a neutron star accretes, its luminosity is dominated by the release of the infalling matter's gravitational potential energy, $L_A \approx 190\,\mathrm{MeV}(\dot{M}/m_b)$, where m_b is the average nucleon mass. Nuclear burning (either steady or via type I X-ray bursts) of the accreted hydrogen and helium generates an additional $\sim 5\,\mathrm{MeV}$ per accreted nucleon. Most of this heat is promptly radiated away, however, and no more than a few percent diffuses inward to heat the interior (Fujimoto et al. 1984, 1987). Nuclear reactions in the deep crust (at $\rho \gtrsim 5\times 10^{11}\,\mathrm{g\,cm^{-3}}$) release about 1 MeV per accreted nucleon (Sato 1979; Blaes et al. 1990; Haensel & Zdunik 1990) and heat the crust directly (Brown & Bildsten 1998; Brown 2000).

In addition to the crustal reactions, the viscous dissipation of r-modes constitutes another heat source in the neutron star's core. For a fiducial neutron star with $M = 1.4M_\odot$ and $R = 10\,\mathrm{km}$, equation (6) implies that $W_d/L_A = 0.046(\nu_{\mathrm{spin}}/300\,\mathrm{Hz})$, or

$$W_d \approx 8.9\,\mathrm{MeV}\left(\frac{\dot{M}}{m_b}\right)\left(\frac{\nu_{\mathrm{spin}}}{300\,\mathrm{Hz}}\right). \tag{7}$$

This heating is very substantial, as it is much greater than the amount of nuclear heating from the crustal reactions. The prospects for detecting the effect of core r-mode heating in *steadily* accreting neutron stars are dim, unfortunately, as it is dwarfed by the accretion luminosity (which is a factor of 20 brighter). The thermal emission from the neutron star is directly observable, however, if accretion periodically halts, as in the neutron star transients (the cooling timescale of the heated core is $\sim 10^4\,\mathrm{yr}$). While continued accretion at low levels between outbursts may contribute some of the quiescent luminosity (see Brown et al. 1998 for a discussion), the thermal emission from the hot crust of the neutron star is impossible to hide, and so observations of L_q set an upper limit on the core temperature. The neutron stars in soft X-ray transients therefore offer the best prospects to look for evidence of viscous heating. In the next section we predict the quiescent luminosity L_q that arises because of the r-mode heating and compare it to the observed luminosities of several neutron star transients.

3. THE QUIESCENT LUMINOSITIES OF NEUTRON STAR TRANSIENTS

The neutron star accretes fitfully, so the spin period and the r-mode amplitude oscillate about the equilibrium defined by the time-averaged accretion rate, $\langle \dot{M}\rangle \equiv t_r^{-1}\int \dot{M}\,dt$, where t_r is the recurrence interval. Moreover, the timescale for viscous dissipation to heat the core is

$$t_H \sim \frac{c_p T}{\langle W_d\rangle}\frac{M}{m_b} \approx 6\times 10^4\,\mathrm{yr}\left(\frac{10^{-11}\,M_\odot\,\mathrm{yr}^{-1}}{\langle \dot{M}\rangle}\right), \tag{8}$$

where c_p is the specific heat per baryon and $\langle W_d\rangle$ is the viscous heating averaged over an outburst/quiescent cycle. Because t_H is much longer than the outburst recurrence time (typically of order years to decades), the core should remain fixed at the temperature set by the balance (over many outburst/quiescent cycles) between heating and cooling processes. We may therefore compute the viscous dissipation using $\langle \dot{M}\rangle$. Some simple estimates of the equilibrium core temperatures and the resulting quiescent luminosities, for when both radiative and neutrino cooling are important, are presented first (§ 3.1). This is followed, in § 3.2, by detailed numerical calculations of the neutron star's thermal structure and a comparison (§ 3.3) to observations of several neutron star transients.

3.1. *Simple Estimates*

In a thermal steady state, the neutron star interior is cooled both by neutrinos emitted from the core and crust and by photons emitted from the surface. To begin, we estimate the luminosity and the equilibrium core temperature set by balancing the heat deposited during an outburst/recurrence cycle, $\langle W_d\rangle$, with each cooling mechanism individually. First, if neutrino emission from the core is negligible (e.g., if the core is superfluid and the Urca processes are exponentially suppressed), then all of the heat generated by viscous dissipation, $\langle W_d\rangle$, is conducted to the surface of the neutron star and escapes as thermal

radiation during quiescence. For the interior to be in a thermal steady state, the quiescent luminosity must then be

$$L_q \approx \langle W_d \rangle = 5.4\times 10^{33}\,\mathrm{erg\,s^{-1}} \left(\frac{\langle \dot{M}\rangle}{10^{-11}\,M_\odot\,\mathrm{yr^{-1}}}\right)\left(\frac{\nu_{\mathrm{spin}}}{300\,\mathrm{Hz}}\right). \tag{9}$$

This estimate depends only on the assumption that neutrino emission is suppressed, and is independent of the crust microphysics.

As a check, we estimate the temperature of the neutron star core. In quiescence the atmosphere and crust come to resemble a cooling neutron star (Bildsten & Brown 1997; Brown et al. 1998). For the temperature increase through the atmosphere and upper crust, we use the fit of Gudmundsson, Pethick, & Epstein (1983),

$$L_\gamma \approx 8.2\times 10^{32}\,\mathrm{erg\,s^{-1}} \left(\frac{T_b}{10^8\,\mathrm{K}}\right)^{2.2}, \tag{10}$$

where T_b is the temperature at a fiducial boundary $\rho_b = 10^{10}\,\mathrm{g\,cm^{-3}}$. Equating L_γ with L_q from equation (9) gives an estimate of the temperature in the upper crust,

$$T_b \approx 2.4\times 10^{8}\,\mathrm{K} \left(\frac{\langle \dot{M}\rangle}{10^{-11}\,M_\odot\,\mathrm{yr^{-1}}}\right)^{0.45}\left(\frac{\nu_{\mathrm{spin}}}{300\,\mathrm{Hz}}\right)^{0.45}. \tag{11}$$

To relate T_b to the core temperature T_c, we use approximate analytic expressions (Brown 2000; eqs. [22] and [23]) for the crust temperature to obtain

$$\left(\frac{T_c}{10^8\,\mathrm{K}}\right)^2 \approx \left(\frac{T_b}{10^8\,\mathrm{K}}\right)^2 + 4.9\left(\frac{L_q}{10^{34}\,\mathrm{erg\,sec^{-1}}}\right), \tag{12}$$

where we have neglected the luminosity due to crustal nuclear reactions. Substituting from equation (11) for T_b, we obtain the core temperature in the absence of neutrino emission,

$$T_c \approx 2.9\times 10^{8}\,\mathrm{K} \left(\frac{\langle \dot{M}\rangle}{10^{-11}\,M_\odot\,\mathrm{yr^{-1}}}\right)^{0.45}\left(\frac{\nu_{\mathrm{spin}}}{300\,\mathrm{Hz}}\right)^{0.45}, \tag{13}$$

where the scalings for $\langle \dot{M}\rangle$ and ν_{spin} are obtained by dropping the second term on the right in equation (12). This estimate agrees quite well with the detailed calculations described in § 3.2.

The core neutrino emissivity is, for modified Urca processes (Shapiro & Teukolsky 1983), $L_\nu^{\mathrm{Urca}} \approx 7.4\times 10^{31}(T_c/10^8\,\mathrm{K})^8$, multiplied by a superfluid reduction factor that goes roughly as $\exp(-\Delta/kT_c)$, where Δ is the superfluid gap energy (Yakovlev & Levenfish 1995). For $\Delta > kT_c$ the net Urca neutrino luminosity is much less than L_q, so that equation (9) is self-consistent. Neutrino emission from crust neutrino bremsstrahlung (Kaminker et al. 1999) at the temperature T_b (eq. [11]) is also not significant, although at higher $\langle \dot{M}\rangle$ it is competitive with radiative cooling. Hence for accretion rates typical of neutron star transients, the majority of the deposited heat is conducted to the surface, and equation (9) provides a robust estimate of the radiative luminosity of the star.

Alternatively, if core neutrino emission is not suppressed (i.e., the nucleons are not superfluid), then modified Urca processes are the dominant coolant and $L_\nu^{\mathrm{Urca}} \approx W_d$. In this case the core temperature is

$$T_c \approx 1.7\times 10^{8}\,\mathrm{K} \left(\frac{\langle \dot{M}\rangle}{10^{-11}\,M_\odot\,\mathrm{yr^{-1}}}\right)^{1/8}\left(\frac{\nu_{\mathrm{spin}}}{300\,\mathrm{Hz}}\right)^{1/8}, \tag{14}$$

and is smaller than if the core were superfluid. A colder core implies a dimmer thermal luminosity from the surface. In order to estimate L_q, we write $W_d = L_q + L_\nu^{\mathrm{Urca}}(T_c)$, where $L_q = L_\gamma(T_b)$, and T_b is related to T_c by equation (12). Under the assumption that $L_q \ll L_\nu^{\mathrm{Urca}}$, the solution of the resulting transcendental equation is

$$L_q \approx 1.8\times 10^{33}\,\mathrm{erg\,s^{-1}} \left(\frac{\langle \dot{M}\rangle}{10^{-11}\,M_\odot\,\mathrm{yr^{-1}}}\right)^{0.3}\left(\frac{\nu_{\mathrm{spin}}}{300\,\mathrm{Hz}}\right)^{0.3}, \tag{15}$$

where we obtain the scalings for $\langle \dot{M}\rangle$ and ν_{spin} by dropping the second term on the right-hand side of equation (12). In this case L_q is less than W_d and L_ν^{Urca}, so our assumption that core neutrino emission is the dominant coolant is self-consistent.

These estimates neglect cooling from other neutrino-producing mechanisms, such as neutrino bremsstrahlung in the crust. Moreover, the core neutrino emissivity depends on the local proper temperature, which increases towards the center of the star because of the gravitational redshift. We now describe our detailed calculations, which take these effects into account.

3.2. *Numerical Calculations*

To calculate the expected quiescent luminosities of accreting neutron stars, we compute hydrostatic neutron star models by integrating the post-Newtonian stellar structure equations (Thorne 1977) for the radius r, gravitational mass m, potential, and pressure with the equation of state AV18+δv+UIX* (Akmal, Pandharipande, & Ravenhall 1998), as described in Brown (2000). With the hydrostatic structure specified, the luminosity L and temperature T are found by solving the entropy and flux equations (Thorne 1977),

$$e^{-2\Phi/c^2}\frac{\partial}{\partial r}\left(Le^{2\Phi/c^2}\right) - 4\pi r^2 n\left(\epsilon_r - \epsilon_\nu\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1/2} = 0 \tag{16}$$

$$e^{-\Phi/c^2}K\frac{\partial}{\partial r}\left(Te^{\Phi/c^2}\right) + \frac{L}{4\pi r^2}\left(1-\frac{2Gm}{rc^2}\right)^{-1/2} = 0 \tag{17}$$

Here ϵ_r and ϵ_ν are the nuclear heating and neutrino emissivity per baryon, n is the baryon density, and K is the thermal conductivity. The potential Φ appears in the time-time component of the metric as e^{Φ/c^2} (it governs the redshift of photons and neutrinos; Misner, Thorne, & Wheeler 1973). We neglect in equation (16) terms arising from compressional heating, as they are of order $T\Delta s(\dot{M}/M)$ (Fujimoto & Sugimoto 1982), s being the specific entropy, and are negligible throughout the degenerate crust and core (Brown & Bildsten 1998). We do not include heating from nuclear reactions in the deep crust. This has the effect of underestimating slightly (by $\lesssim 10\%$) the quiescent luminosity of the neutron star transient. Equations (16) and (17) are integrated outwards to a density $\rho_b = 10^{10}\,\mathrm{g\,cm^{-3}}$. We there impose a boundary condition relating L and T with the fitting formula of Potekhin, Chabrier, & Yakovlev (1997) for a partially accreted crust. By incorporating a parameter describing the depth of a light element (H and He) layer, this formula differs from that of Gudmundsson et al. (1983), which we used for our simple estimates (§ 3.1). We set the depth of this light element layer to where the density is $\approx 10^5\,\mathrm{g\,cm^{-3}}$, which is roughly where the accreted material burns to heavier elements (Hanawa & Fujimoto 1986).

The high thermal conductivity of the neutron star's core insures that it is very nearly isothermal, regardless of the detailed dependence of the heating rate ϵ_r on the radius. For the core temperatures typical of LMXBs, bulk viscosity is unimportant, so we assume that the heating is from ordinary shear viscosity. The rate per unit volume is just $2\eta\delta\sigma^{ab}\delta\sigma^{*}_{ab}$, where η is the shear viscosity and $\delta\sigma_{ab}$ is the kinematic shear. If we neglect the dependence of shear viscosity on density (since the density is approximately constant in the neutron star's core), this rate is just proportional to r^2 for an $(l = 2, m = 2)$ r-mode (Lindblom et al. 1998). Hence we take $\epsilon_r \propto r^2$, and normalize it so that the heating rate, when integrated over the core, satisfies equation (6).

The microphysics used to integrate equations (16) and (17) is fully described in Brown (2000), so here we just highlight two modeling uncertainties. First, standard calculations presume that the neutron star's crust is a pure lattice, and hence the conductivity is dominated by electron-phonon scattering. Over the lifetime of an LMXB, however, the neutron star can easily accrete enough matter to replace its entire crust (requiring about $0.01 M_\odot$). The accreted crust is formed from the products of hydrogen and helium burning and is likely to be very impure (Schatz et al. 1999). A lower conductivity from impurities lowers the surface temperature, and hence the quiescent luminosity, for a given core temperature. We model the low thermal conductivity of a very impure crust by using electron-ion scattering (Haensel, Kaminker, & Yakovlev 1996) throughout the crust.

The second modeling uncertainty is the superfluid transition temperatures, for which estimates vary widely (see Tsuruta 1998, and references therein). When the core temperature is much less than the superfluid transition temperature, emissivity from Cooper pairing is unimportant (Yakovlev, Kaminker, & Levenfish 1999) and the superfluidity suppresses the neutrino emission by roughly the Boltzmann factor $\exp(-\Delta/k_B T)$, for a superfluid gap energy Δ. We perform our calculations for two models, one with superfluidity parameterized as in Brown (2000), with a typical gap energy $\Delta \sim 0.5\,\mathrm{MeV}$, and another model with a normal core, $\Delta = 0$.

Figure 1 demonstrates the thermal structure of such a neutron star with a time-averaged accretion rate $\langle\dot{M}\rangle = 2.4 \times 10^{-11}\,M_\odot\,\mathrm{yr}^{-1}$ (the rate inferred for Aql X-1) for a spin frequency of 275 Hz (*solid lines*) and 549 Hz (*dotted lines*). If the core is superfluid (the upper pair of curves), then the neutrino luminosity from crust bremsstrahlung (region leftward of the vertical dot-dashed line) is roughly comparable to the photon luminosity. In contrast, if the core were normal (so that the modified Urca processes were unsuppressed) but the viscosity remained independent of temperature (so that a thermal steady state could be reached), then only about 10% of the heat generated in the core would be conducted to the surface. The rest of the heat is balanced by modified Urca neutrino emission. The core temperature and fraction of viscous heat conducted to the surface compare well with the estimates in § 3.1.

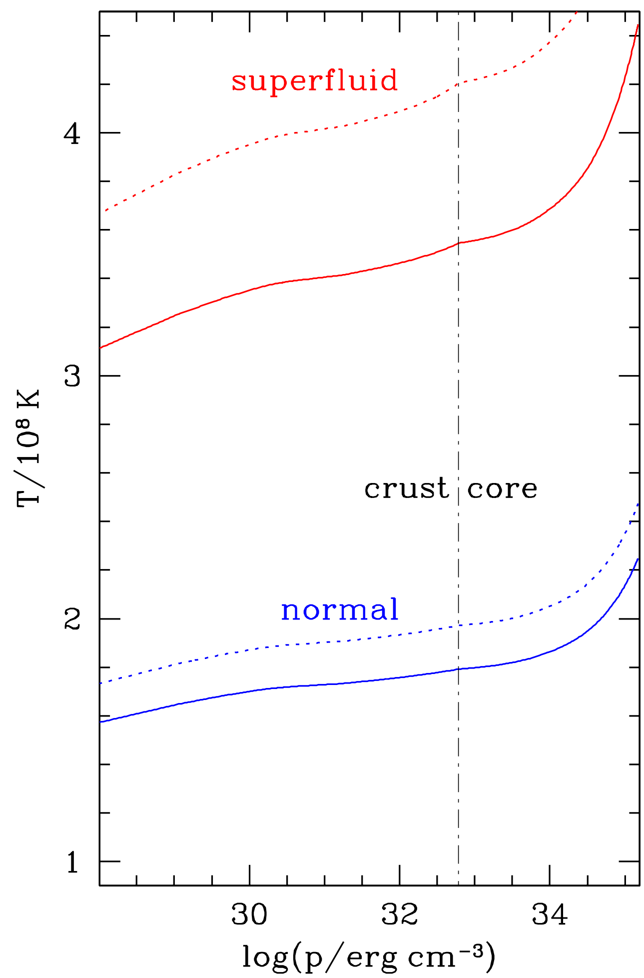

FIG. 1— The thermal structure of a neutron star accreting at a time-averaged rate of $2.4 \times 10^{-11}\,M_\odot\,\mathrm{yr}^{-1}$ (e.g., Aql X-1), for two different spin frequencies: 275 Hz (*solid lines*) and 549 Hz (*dotted lines*). The upper pair of curves are for a superfluid core (region rightward of the vertical dot-dashed line); the lower pair, for a normal core.

3.3. *Comparison to Observed Transients*

A superfluid core is cooled mainly by conduction of heat to the surface, at least until the interior temperature is high enough to activate crust neutrino bremsstrahlung. Figure 2 shows the expected quiescent luminosity L_q as a function of $\langle\dot{M}\rangle$ for this case, with a range (*shaded region*) of rotation frequencies $200\,\mathrm{Hz} < f < 600\,\mathrm{Hz}$. The inferred $\langle\dot{M}\rangle$ and L_q for several neutron star transients are also plotted (*squares*) for comparison. With the exception of EXO 0748-676[2], the neutron star transients with measured quiescent luminosities are too dim, by a factor of 5–10, to be consistent with viscous heating of the magnitude assumed here. We must conclude, then, that *either the accretion torque is much less than* $\langle\dot{M}\rangle(GMR)^{1/2}$, *or that a steady-state r-mode does not set their spin.*

The quiescent luminosities for Aql X-1, Cen X-4, and 4U 1608–522 use the bolometric corrections appropriate for a H atmosphere spectrum (Rutledge et al. 1999a); L_q for the Rapid Burster is from Asai et al. (1996a). We infer the time-averaged accretion rate from $\langle\dot{M}\rangle \approx (t_o/t_r)(L_o/GMR^{-1})$, where

[2]EXO 0748-676 is likely to accrete during quiescence, as suggested by observations (with *ASCA*) of variability on timescales $\gtrsim 1000\,\mathrm{s}$ (Corbet et al. 1994; Thomas et al. 1997).

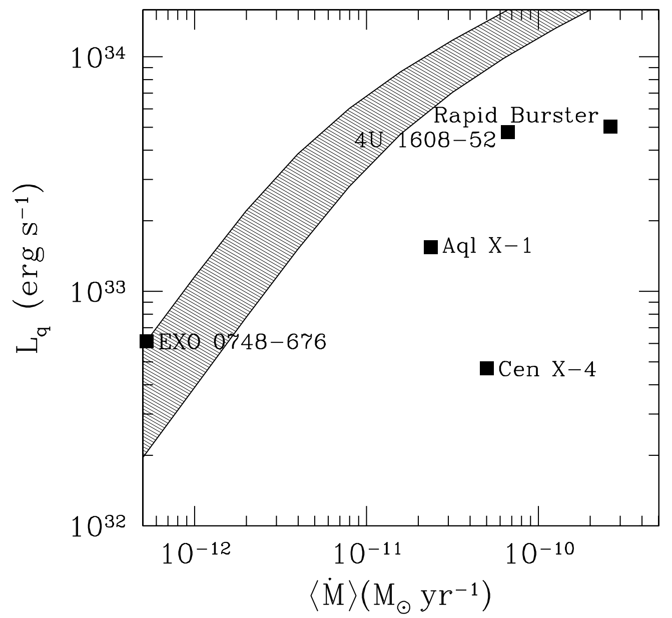

FIG 2.— Quiescent luminosities as a function of time-averaged accretion rate $\langle\dot{M}\rangle$. The heating from viscous dissipation of the r-mode is from eq. (6), and the neutrino emission from the core is suppressed by nucleon superfluidity. The shaded region corresponds to rotation frequencies between 200 Hz (*lower curve*) and 600 Hz (*upper curve*). Neutrino cooling from crust bremsstrahlung is important rightward (i.e., at higher $\langle\dot{M}\rangle$) of the knee in the shaded region. Also shown are the inferred quiescent luminosities and time-averaged accretion rates for several neutron star transients.

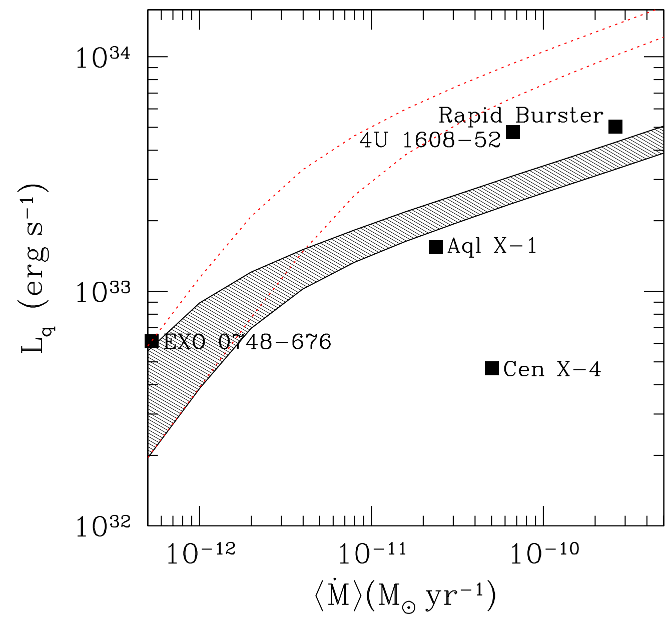

FIG. 3— The same as Fig. 2, but for a normal core. We also show (*thin dotted lines*) $L_q(\langle\dot{M}\rangle)$ for a neutron star with a crust of light elements at $\rho < 10^{10}\,\mathrm{g\,cm^{-3}}$.

t_o and L_o are the outburst duration and luminosity and the distances are taken from Chen, Shrader, & Livio (1997).[3] Outburst fluences for Aql X-1 and the Rapid Burster are accurately known (*RXTE*/All-Sky Monitor public data); for the remaining sources $\langle\dot{M}\rangle$ is estimated from peak luminosities and outburst rise and decay timescales (Chen et al. 1997).

Our estimates for $\langle\dot{M}\rangle$ depend on the inferred source distance. When most of the r-mode heating W_d is conducted to the surface, however, as in the superfluid core case for $\langle\dot{M}\rangle \lesssim 10^{-11}\,M_\odot\,\mathrm{yr}^{-1}$, the predicted quiescent luminosity is $L_q \approx W_d \propto \langle\dot{M}\rangle$ (see eqs. [6] and [7]), and hence depends on the source distance in the same way as does $\langle\dot{M}\rangle$. Therefore, our comparison of L_q predicted from r-mode heating and the quiescent luminosity actually observed is *independent* of distance. In this regime, the relation between L_q and $\langle\dot{M}\rangle$ *is also independent of the microphysics in the crust.*

As shown by Levin (1999), the temperature dependence of viscosity in a normal fluid likely prevents a steady-state r-mode. For comparison, however, we plot in Figure 3 the case where modified Urca neutrino emission from the core is allowed (as it would be in a normal fluid) but the r-mode amplitude is steady, i.e., we assume that a thermogravitational runaway has somehow been avoided. As a result, neutrino emission efficiently cools the core, and so the radiative luminosity L_q is less for a given $\langle\dot{M}\rangle$. For this case, with the exception of Cen X-4, the neutron star transients have quiescent luminosities roughly consistent with that predicted. Because there is a characteristic core temperature, namely, that at which neutrino cooling equals radiative cooling, the relation between $\langle\dot{M}\rangle$ and L_q is no longer independent of distance, unlike the case shown in Figure 2. The knee in the shaded region is where the neutrino and photon luminosities are comparable. Rightward of this knee ($\langle\dot{M}\rangle \gtrsim 10^{-12}\,M_\odot\,\mathrm{yr}^{-1}$) neutrino cooling prevents the core temperature, and hence the photon luminosity, from rising rapidly with increasing $\langle\dot{M}\rangle$. Should the crust have a higher conductivity (e.g., if it were more pure) than we have assumed here, then the shaded region rightward of the knee would move upwards, i.e., the predicted L_q would be even higher. To illustrate this we computed $L_q(\langle\dot{M}\rangle)$ using the $L_\gamma(T_b)$ relation for a crust composed of light elements (and having a higher conductivity) for densities less than $\rho_b = 10^{10}\,\mathrm{g\,cm^{-3}}$ (*dotted lines*).

It should be noted that the actual thermal radiation from a neutron star's surface is in general *less* than the observed quiescent luminosity, since other emission mechanisms are possible, such as accretion via a low-efficiency advective flow (Narayan, McClintock, & Yi 1996) or magnetospheric emission (Campana et al. 1998a). Evidence for other, non-thermal emission processes are the hard power-law tails observed from Cen X-4 (*ASCA*; Asai et al. 1996b) and Aql X-1 (*BeppoSAX*; Campana et al. 1998b). In addition, variability on timescales of a few days has been observed from Cen X-4 (van Paradijs et al. 1987; Campana et al. 1997). As a result, a plot showing thermal emission (as opposed to observed L_q) would have the data points shifted downward in Figures 2 and 3. In other words, the quiescent luminosity inferred from observations is likely to overestimate the actual thermal emission from the neutron star. This strengthens our conclusion regarding the incompatibility of steady-state r-mode heating with the observations.

There are stronger neutrino emission mechanisms possible than modified Urca and crust bremsstrahlung. Recently, there has been renewed interest in the direct Urca process (Lattimer et al. 1991), which is allowed if the proton fraction exceeds 0.148 or if hyperons are present (Prakash et al. 1992). Other exotic mechanisms may be possible, including pion condensates

[3]Recent observations (Callanan, Filippenko, & Garcia 1999) resolved the optical counterpart of Aql X-1 into two objects. We use the distance estimate (2.5 kpc) of Chevalier et al. (1999), which accounts for the interloper star.

(Umeda et al. 1994), kaon condensates (Brown et al. 1988), or quark matter (Iwamoto 1982). The exotic mechanisms have the same temperature dependence as the direct Urca ($\propto T^6$) but are weaker. Should any of these enhanced processes occur, the core will be much colder, and the heat radiated from the surface much weaker, than in the calculations here. For example, balancing the viscous heating with neutrino emission from a pion condensate, $L_\nu^\pi \approx 2.0 \times 10^{39} (T/10^8\,\mathrm{K})^6\,\mathrm{erg\,s^{-1}}$ (Shapiro & Teukolsky 1983), implies that $T_c \approx 1.2 \times 10^7 (\langle\dot{M}\rangle/10^{-11}\,M_\odot\,\mathrm{yr^{-1}})^{1/6}\,\mathrm{K}$, and, from equations (10) and (12), that $L_q \approx 6.0 \times 10^{30} (\langle\dot{M}\rangle/10^{-11}\,M_\odot\,\mathrm{yr^{-1}})^{0.45}\,\mathrm{erg\,s^{-1}}$. This is much dimmer than that observed. Of course, it is possible that superfluidity reduces L_ν such that the core temperature is just enough to explain the observed quiescent emission. It is difficult, however, to arrange *all* of the sources to obey such a relation.

4. CONCLUSIONS

Using the assumption that the accretion torque is balanced by angular momentum loss from gravitational radiation by an r-mode pulsation of constant amplitude, we find that the expected quiescent luminosities of the neutron star X-ray transients, for rotation rates of 200–600 Hz, are characteristically brighter than those observed. Reconciling the observations with the presence of r-mode heating requires that neutrino emission from the core be unsuppressed, as for a normal core. In this case, however, the r-mode is thermally unstable and cannot remain at a constant amplitude, unless some mechanism prevents a runaway. It therefore seems unlikely that the spin frequency of Aql X-1 is a signature of a steady-state core r-mode pulsation. We note, however, that the same conclusion cannot be drawn for the bright, persistent LMXBs (such as Sco X-1, which could be detected by gravitational wave experiments soon to be operational). Uncertainties in the nuclear burning and the accretion luminosity cannot constrain the surface thermal luminosity to within $\lesssim 5\%$, which is necessary to differentiate the r-mode heating from the accretion luminosity.

In addition to Aql X-1, there is one other neutron star transient which is known to be spinning rapidly, and that is the 401 Hz accreting pulsar (Wijnands & van der Klis 1998) in the transient SAX J1808.4–3658 (in't Zand et al. 1998). This source has not yet been detected in quiescence. Given a recurrence interval of 1.5 yr, an outburst duration of ≈ 20 day, and an outburst accretion rate of $\approx 3 \times 10^{-10}\,M_\odot\,\mathrm{yr^{-1}}$ (in't Zand et al. 1998), we expect a quiescent luminosity $\gtrsim 5.8 \times 10^{33}\,\mathrm{erg\,s^{-1}}$ if the core is superfluid and an active r-mode pulsation balances the accretion torque in this system. This L_q corresponds to an unabsorbed flux (4 kpc distance) of $3 \times 10^{-12}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$, which is about ten times the flux expected if the only heat source were crust nuclear reactions (Brown et al. 1998). Future *ASCA*, *Chandra*, and *XMM* observations will assist in constraining the viscous damping present. The luminosity from the viscous damping is much larger than the expected magnetospheric emission (Becker & Trümper 1997), and so interpretation of the spectrum should be unambiguous in the absence of accretion onto the neutron star's surface.

At $\langle\dot{M}\rangle \lesssim 10^{-11}\,M_\odot\,\mathrm{yr^{-1}}$, all of the viscous heating in the core is radiated from the neutron star's surface during quiescence. As noted in section 3.3, the relation between $L_q(\langle\dot{M}\rangle)$ then depends only on the accretion torque, and not on the source distance and crust microphysics. *Chandra* and *XMM* are ideally suited for a study of a population of low-luminosity neutron stars, which offer excellent prospects for a clean determination of the amount of viscous heating present.

At higher $\langle\dot{M}\rangle$, for which neutrino cooling from the crust contributes to balancing the viscous heating, the quiescent luminosity depends on the crust microphysics. In our calculations we assume that the neutron crust is very impure and hence used thermal conductivity dominated by electron-ion collisions. If the conductivity of the crust is higher than we have assumed (e.g., if the crust is a pure lattice), then the predicted quiescent luminosity L_q would be even higher than that plotted in Figures 2 and 3. In addition, we underestimated the predicted L_q by neglecting the effect of direct heating of the neutron star crust by nuclear reactions occurring near neutron drip (Brown et al. 1998). Moreover, taking into account the possibility that non-thermal emission contributes to the observed quiescent luminosity further widens the gap between the observed L_q and that inferred from the r-mode spin regulation hypothesis. All of these effects further strengthen our conclusions.

If the r-mode is not in steady state, then there remain several possibilities: either the superfluid viscosity is so strong that it suppresses the r-mode instability entirely, or the mode saturation amplitude is so small that it is unimportant at all the spin frequencies observed, or else the neutron star is in a limit cycle (Levin 1999) of spin-up to some critical frequency, followed by rapid spin-down and heating. A detailed study of the spin evolution is necessary to determine if the spin periods of the neutron stars are consistent with such a scenario. In particular, it remains an open question as to whether one should expect to observe a population of slowly spinning neutron stars with low-mass companions, such as Her X-1, 4U 1626–67, and GX 1+4.

The study of r-modes in neutron stars is rapidly evolving in response to the interest aroused in the general relativity community. While this paper was being refereed, several theoretical developments occured that are relevant for this study. First, Lindblom & Mendell (1999) showed that unless the superfluid entrainment parameter assumes a very special value, superfluid mutual friction is not competitive with gravitational radiation for the r-mode amplitude evolution. There is therefore a conflict between theory and experiment: while theoretical calculations show that r-modes in superfluid neutron stars should be excited, the observations discussed in this paper are direct evidence against the r-modes having a sufficient steady amplitude to limit the spin of the neutron star, and the clustering of LMXB spin frequencies argues against an recurrent instability. This contradiction is likely resolved by consideration of the presence of a solid crust (Bildsten & Ushomirsky 1999), which dramatically enhances the dissipation rate and damps the r-modes for typical core temperatures and spin frequencies of LMXBs. The findings presented in this paper lend observational support to that conclusion.

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